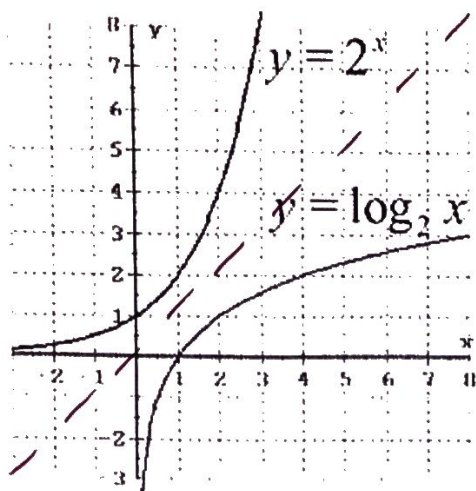


Notes - Logarithmic Function Basics

****Logarithmic functions are the inverse of exponential functions.**

(Like addition and subtraction or multiplication and division. One undoes the other).

ex: $\ln e^3 = 3$, $e^{\ln 6} = 6$



The graph shows that exponentials and logs are inverses, since they are reflections of each other across the line $y=x$.

The graph of a logarithm has the following properties:

Domain: $(0, \infty)$

*You can't take the log of a negative number!! *← or zero!*

Range: $(-\infty, \infty)$

Goes through the point $(1, 0)$

Has no y-intercept

A logarithmic function, $\log_b x$, is read as "log base b of x".

How would you say $\log_5 125$? log base 5 of 125

*If a problem just has "log" with no base number, the base is assumed to be 10.

*If a problem has "ln", this is called the **natural log** and is log base e .

You can rewrite exponential equations in logarithmic form. The base of the exponent is also the base of the logarithm! The exponent goes on the opposite side of the equals sign.

For example, $6^{-2} = \frac{1}{36}$ can be written as $\log_6 \frac{1}{36} = -2$.
(Arrows point from "exponent" to -2 and "base" to 6 in both forms.)

Or $x^z = g$ can be written as $\log_x g = z$.
(Arrows point from "exponent" to z and "base" to x in both forms.)

You try:

1. $3^4 = 81$

$\log_3 81 = 4$

2. $2^x = 16$

$\log_2 16 = x$

3. $2^{-3} = \frac{1}{8}$

$\log_2 \frac{1}{8} = -3$

4. $h^m = r$

$\log_h r = m$

5. $e^x = 2.718$

$\ln 2.718 = x$

Similarly, you can rewrite logarithmic equations in exponential form. The base of the logarithm is also the base of the exponential expression. The exponent is on the opposite side of the equals sign.

For example, $\log_2 8 = 3$ can also be written as $2^3 = 8$.

\swarrow exponent \swarrow exponent
 \uparrow base \uparrow base

Or $y = \log_b x$ is the same as $b^y = x$.

\swarrow exponent \swarrow exponent
 \uparrow base \uparrow base

You try:

1. $\log_4 16 = 2$
 $4^2 = 16$

2. $\log_7 1 = 0$
 $7^0 = 1$

3. $\log_5 x = 4$
 $5^4 = x$

4. $\log_a c = b$
 $a^b = c$

5. $\log 0.1 = -1$
 $10^{-1} = 0.1$

\swarrow log base 10

To evaluate logarithmic expressions or solve logarithmic equations, you can first rewrite in exponential form and then work from there.

For example, to evaluate $\log_4 64$, you can think of it as $\log_4 64 = x$ and rewrite it as $4^x = 64$. Since $4^3 = 64$, the answer is 3.

To solve $\log_2 32 = 3x$, rewrite it as the exponential equation $2^{3x} = 32$. This is just like what we've solved before. I rewrite 32 as a power of 2 to make my bases the same and then set my exponents equal. $2^{3x} = 2^5$ so $3x = 5$ and thus $x = \frac{5}{3} = 1\frac{2}{3}$.

You try:

1. $\log_{100} 100,000 = x$
 $100^x = 100,000$
 $x = 2.5$

2. $\log_5 625 = x$
 $5^x = 625$
 $x = 4$

3. $\log_{25} 5$
 $25^x = 5$
 $5^{2x} = 5^1$ $x = \frac{1}{2}$

4. $\log_6 x = 3$

$6^3 = x$
 $x = 216$

5. $\log_x 1000 = 3$

$x^3 = 1000$
 $x = 10$

6. $\log_4 (5x+1) = 2$

$4^2 = 5x+1$
 $16 = 5x+1$
 $15 = 5x$
 $x = 3$

10-2

Skills Practice

Logarithms and Logarithmic Functions

Write each equation in logarithmic form.

1. $2^3 = 8$

$\log_2 8 = 3$

2. $3^2 = 9$ $\log_3 9 = 2$

3. $8^{-2} = \frac{1}{64}$

$\log_8 \frac{1}{64} = -2$

4. $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$\log_{\frac{1}{3}} \frac{1}{9} = 2$

Write each equation in exponential form.

5. $\log_3 243 = 5$

$3^5 = 243$

6. $\log_4 64 = 3$

$4^3 = 64$

7. $\log_9 3 = \frac{1}{2}$

$9^{\frac{1}{2}} = 3$

8. $\log_5 \frac{1}{25} = -2$

$5^{-2} = \frac{1}{25}$

Evaluate each expression.

9. $\log_5 25$

$5^x = 25$ $x = 2$

11. $\log_{10} 1000$

$10^x = 1000$ $x = 3$

13. $\log_4 \frac{1}{64}$

$4^x = \frac{1}{64}$ $x = -3$

15. $\log_8 8^3$

$8^x = 8^3$ $x = 3$

10. $\log_9 3$

$9^x = 3$ $3^{2x} = 3^1$ $x = \frac{1}{2}$

12. $\log_{125} 5$

$125^x = 5$ $5^{3x} = 5^1$ $x = \frac{1}{3}$

14. $\log_5 \frac{1}{625}$

$5^x = \frac{1}{625}$ $5^x = 5^{-4}$ $x = -4$

16. $\log_{27} \frac{1}{3}$

$27^x = \frac{1}{3}$ $3^{3x} = 3^{-1}$ $x = -\frac{1}{3}$

Solve each equation or inequality. Check your solutions.

17. $\log_3 x = 5$

$3^5 = x$ $x = 243$

~~19. $\log_4 y < 0$~~

18. $\log_2 x = 3$

$2^3 = x$ $x = 8$

20. $\log_{\frac{1}{4}} x = 3$

$\left(\frac{1}{4}\right)^3 = x$ $\left(\frac{1}{64}\right)^1 = x$ $x = \frac{1}{64}$

22. $\log_b 3 = \frac{1}{2}$

$b^{\frac{1}{2}} = 3$ $b = 9$

24. $\log_2 (4x - 4) > 5$

~~26. $\log_6 (3y - 5) \geq \log_6 (2y + 3)$~~

23. $\log_6 (4x + 12) = 2$

$6^2 = 4x + 12$ $36 = 4x + 12$ $24 = 4x$ $x = 6$

25. $\log_3 (x + 2) = \log_3 (3x)$

$x + 2 = 3x$

$x = 1$

Evaluating Logarithms

Evaluate each expression.

1) $\log_7 \frac{1}{343}$

2) $\log_2 \frac{1}{8}$

3) $\log_4 16$

4) $\log_3 27$

5) $\log_7 49$

6) $\log_2 4$

7) $\log_7 \frac{1}{49}$

8) $\log_{64} \frac{1}{4}$

9) $\log_6 36$

10) $\log_6 \frac{1}{216}$

11) $\log_5 125$

12) $\log_4 \frac{1}{64}$